

## Rethinking the Concept of Space-Time in the General Theory of Relativity: The Deflection of Starlight and the Gravitational Red Shift

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In order to confirm the prediction of the General Theory of Relativity that space and time are relative and that matter warps a dynamic space-time continuum that surrounds it, Einstein suggested two optical tests: one being the gravitational deflection of starlight, and the second being the gravitational red shift. The tests confirmed the predictions of the General Theory of Relativity. The observational results for the deflection of starlight were inconsistent with the predictions of Newtonian theory as developed by Soldner, and the Newtonian theory developed by Laplace was not rigorous enough to account for the observational results of the gravitational red shift. Both observational results can be explained equally well by (1) the General Theory of Relativity, which assumes that photons submissively propagate through a dynamic space-time continuum, which is warped by the presence of matter; or (2) the theory presented here, which assumes that the photon itself has dynamic properties and it propagates through absolute Euclidean space and Newtonian time. The second alternative, which can explain the deflection of starlight, the gravitational red shift, gravitational lensing, and clock synchronization in the Global Positioning System (GPS) has the advantage of encompassing many of the dynamical properties of photons that were neither known to Newton nor employed by Einstein.

### 1. Introduction: Two Optical Phenomena that are Influenced by Gravity

According to the General Theory of Relativity, gravity influences both the motion and the spectral properties of light, not by acting on light itself, which is considered to be composed of point-like photons, but by directly acting on a dynamic space-time continuum through which the light submissively propagates. After giving a short introduction to these two optical phenomena, the first of which is considered to be the *experimentum crucis* in favor of the General Theory of Relativity, I will present an alternative explanation of the effect of gravity on light based on some known and proposed dynamical properties of the photon. In essence, I put the mechanics back into the quantum mechanics of light and take the mechanics out of the description of space and time. This approach has already allowed me to use the dynamic properties of light to describe and explain why charged particles and/or particles with a magnetic moment cannot exceed the speed of light—without assuming the relativity of space and time posited by the Special Theory of Relativity [1].

Gravity affects the motion of light, which Isaac Newton [2] may have been thinking about when he asked, “*Do not Bodies act upon Light at a distance, and by their action bend its Rays; and is not this action (caeteris paribus) strongest at the least distance?*” Subsequently, Johann Georg von Soldner calculated that the deflection of starlight by

the sun would amount to 0.84 arcseconds<sup>1</sup> by treating “*a light ray as a heavy body*” and using Newton’s Law of Universal Gravitation [4-12]. At the time, the predicted deflection was technically unobservable and the paper was forgotten. Approximately a century later, Albert Einstein [13,14], in his initial development of the General Theory of Relativity, assumed that gravity did not influence light itself, but the space-time continuum through which the light propagated. According to Einstein, the speed of light would slow as the starlight propagated through the gravitational field of a massive body and this slowing, according to Huygens’ Principle, would result in the bending of light. Einstein first calculated that the starlight propagating through the gravitational field of the sun would be deflected by 0.83 arcseconds. However, after Einstein [15,16] completed the General Theory of Relativity, he deduced that a gravitational mass would also cause a curvature of space itself, and he doubled the predicted magnitude of the deflection of starlight by the sun to 1.7 arcseconds. While Newton and Soldner considered gravity to act dynamically on light propagating through absolute space and time, Einstein considered matter to warp a dynamic and relative space-time continuum so that the apparent force of gravity was actually a result of the action

<sup>1</sup> Note that small differences in the values of the deflection result from historical differences in the values of the constants used to derive the magnitude of the deflection [3].

of matter on the geometry of space-time through which the point-like photons that made up the light submissively propagated.

Realizing that he could use the deflection of starlight by the sun to test the veracity of the logical, elegant, and beautiful equations of the General Theory of Relativity that predicted the influence of matter on the curvature of space-time [17-20], Arthur Eddington helped organize expeditions that would test Einstein's General Theory of Relativity by observing the position of the stars during a total eclipse of the sun (Fig. 1). In such a test, the positions of the stars in the field near the sun that would be visible during a solar eclipse would be compared with the positions of the same stars observed at night, at a different time of year, when the sun's gravity no longer influenced the starlight traveling from the stars to the earth. *Caeteris paribus*, the difference in the positions of the stars would be attributable to the gravitational deflection of starlight [21].

The observational results were inconsistent with the "single deflection" predicted by Newtonian theory as developed by Soldner and supported the "double deflection" predicted by Einstein's General Theory of Relativity [22-28], and since then, the gravitational bending of light by the sun has been and is still considered to be one of the crucial observations in support of the assumption that the space-time continuum is dynamic and warped by matter as posited by Einstein's General Theory of Relativity. R. J. Trumpler [29] wrote, "*No other theory is at present able to account for the numerical values of the observed displacements. The assumption that there is an actual curvature of space in the immediate surroundings of the Sun, which is implied in Einstein's theory, seems indeed to furnish the only satisfactory explanation why the observed light deflections are twice as large as those predicted on the basis of Newton's theory.*" By taking into consideration dynamical properties of light unknown to Newton and not employed by Einstein, I can explain the observed "double deflection" of starlight without invoking a dynamic space-time continuum that can be warped by the presence of matter.

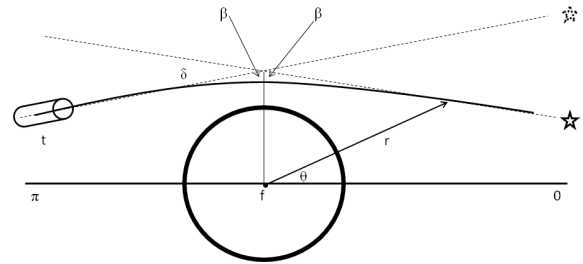


Fig.1: The deflection of starlight. As a result of the gravitational attraction of the sun, starlight composed of photons is deflected as it passes close to the sun. Consequently, the source (solid star) of the starlight appears to be displaced by angle  $\delta$  to the position marked by the dotted star. The predicted magnitude of the deflection depends on the assumptions concerning the nature of light, time and space. Angle  $\delta$  is determined from solving the equation for a hyperbola for  $\beta$ , which is related to the eccentricity ( $\epsilon$ ) of the hyperbolic trajectory, and which is obtained as a term in the equation of motion of the trajectory of starlight through the gravitational field of the sun. The solid line extending from the actual position of the star to the telescope is described by the equation of motion of the trajectory of starlight which gives the dependence of  $r$  upon  $\theta$ . The predicted trajectories, evaluated from 0 to  $\pi$  radians, are given by Eqns. (22) and (A14).

Gravity also affects the spectral properties of light. The spectral properties of light were first revealed by Isaac Newton [30] when he used a prism to show that a beam of sunlight was composed of a continuous spectrum of colors. In order to understand how light emanating from stars would be influenced by gravity, Joseph Priestley [31], John Michell [32], and Pierre-Simon Laplace [33] postulated that the particles of light emanating from a fixed star would be gravitationally attracted to the star according to Newton's Law of Universal Gravitation in the same manner that any other body with a *vis inertiae*, or inertial mass, would be attracted to a gravitational mass. Consequently, they surmised that the velocity of light emitted by a star would be diminished by the gravitational attraction.

In order to explain the effect of gravity on light, Albert Einstein [13,15,34,35,36] extended the principle of relativity that he used to describe uniform motion in his Special Theory of Relativity to accelerated motion. According to Einstein [15], the period of time ( $\sigma$ ) in a gravitational potential ( $\Phi = -\frac{GM}{r}$ ) was related to the period of time ( $\tau$ ) in a reference system at  $r = \infty$  by the following equation:

$$\sigma = \tau \left(1 - \frac{\Phi}{c^2}\right) = \tau \left(1 + \frac{GM}{rc^2}\right) \quad (1)$$

Einstein [15] concluded that “*the clock goes more slowly if set up in the neighbourhood of ponderable masses. From this it follows that the spectral lines of light reaching us from the surface of large stars must appear displaced towards the red end of the spectrum.*”

In the next section I will discuss some of the known and reasonably proposed dynamical properties of photons [37]. By taking into consideration these dynamical properties, I provide an alternative explanation for the influence of matter on the motion and spectral properties of light that do not depend on the warping of a dynamic space-time continuum by matter as postulated by the General Theory of Relativity.

## 2. Results

### 2.1. The dynamical properties of photons

The existence of radiation pressure [38] is an indication that photons are dynamic entities and that they carry linear momentum. The magnitude of the linear momentum of a photon depends on the wavelength or frequency of the photon and according to quantum theory is given by  $\frac{h}{\lambda}$  [39] or  $\frac{h\nu}{c}$  [40]. Classical physics also entertains the possibility that light possesses angular momentum [41]. Angular momentum was originally known as the moment of momentum, which emphasized the importance of a radial extension. The spin angular momentum for each and every photon is equal to  $\frac{h}{2\pi}$  [42-51]. Interestingly, the spin angular momentum is unique in that it is the only property shared by all photons, independent of their frequency and wavelength.

John Nicholson [52,53] introduced the importance of angular momentum in understanding the characteristic spectrum of atoms, and interpreted Planck's constant, as a “*natural unit of angular momentum* [54],” indicating that angular momentum might be quantized and “*the angular momentum of an atom can only rise or fall by discrete amounts when electrons leave or return.*” Niels Bohr [55] applied Nicholson's idea of quantized angular momentum to Rutherford's planetary model of the atom [56], and wrote “*In any molecular system consisting of positive nuclei and electrons in which the nuclei are at rest relative to each other and the electrons move in circular orbits, the angular momentum of every electron round the centre of its orbit will in the*

*permanent state of the system be equal to  $h/2\pi$ , where  $h$  is Planck's constant.*” According to Arnold Sommerfeld [57], “*...in the process of emission..., we demanded...the conservation of energy. The energy that is made available by the atom should be entirely accounted for in the energy of radiation  $\nu$ , which is, according to the quantum theory of the oscillator, equal to  $h\nu$ . With the same right, we now demand the conservation of momentum and of moment of momentum: if in a change of configuration of the atom, its momentum or moment of momentum alters, then these quantities are to be reproduced entirely and unweakened in the momentum and moment of momentum of the radiation.*”

The spin angular momentum of photons is basic for understanding the selection rules that describe the atomic spectra for emission and absorption that demand conservation of angular momentum between a photon and the atomic absorber or emitter [58-62]. Thus, each photon carries both linear momentum and spin angular momentum, which can be either parallel or antiparallel to its direction of motion [63]. Angular momentum is one of the fundamental concepts of physics [64-66], and if indeed, a photon has extension in the radial direction, as suggested by Lorentz [67] and Millikan [68], in order to explain interference phenomena; and Wayne [1,37,69], in order to explain the observed arrow of time and why charged particles cannot exceed the speed of light, then spin angular momentum will represent rotational motion of or within the photon.

I propose that the existence of the spin angular momentum of a photon is an indication of the potential, for a general theory of optical phenomena, to consider the rotational motion of a photon in addition to its translational motion. This opportunity is analogous to the one seized initially by Rudolf Clausius [70] who provided an explanation of the observed values of specific heats by treating molecules as having both translational and rotational motions. Prior to Clausius' consideration of rotational motion, the mechanical theory of heat [71-73], which only took into consideration the translational motions of molecules, could not describe the observed specific heats of diatomic gases. The inclusion of rotational motion brought the predictions of the mechanical theory of heat closer to the observed values. James Clerk Maxwell [74] further utilized the concept of the equipartition of energy when he asserted that in ideal gases, the energy of rotation was equal to the energy of translation. In his study of the specific heat of solids, Ludwig Boltzmann [75-77]

generalized the equipartition theorem to say that the average energy of all systems was equally divided among all the independent components of motion, including the potential and kinetic energies of oscillators. Lord Rayleigh and James Jeans extended the equipartition theorem to describe the distribution and polarization of black body radiation [78-84].

The total energy ( $h\nu$ ) of a photon can be transferred to or from an atom when the photon is destroyed or created upon absorption or emission, respectively. In optical processes that do not depend on absorption, it is possible that only parts of the total energy may be relevant in describing and explaining the phenomenon. I consider the photon in free space to be an adiabatic thermodynamic system composed of a longitudinal oscillator, containing potential and kinetic energy and a rotational oscillator, containing potential and kinetic energy [37]. The two orthogonal oscillators are in thermal equilibrium and, by extension of the equipartition theory; the total energy of the photon is equally distributed among the four degrees of freedom.

By using the equipartition theorem and taking the assumed rotational as well as the translational properties of the photon propagating through absolute Euclidean space and Newtonian time into consideration when deriving the equation of motion, in the next section I will show that the observed magnitude of the gravitational deflection of starlight, which was the *experimentum crucis* in favor of the General Theory of Relativity, can be described and explained without invoking the General Theory of Relativity that posits that matter induces a curvature of a dynamical space-time continuum that results in the hyperbolic trajectory that starlight takes around the sun. The ability to describe and explain the observed “double deflection” of starlight lends support to the validity of the complex, dynamical model of the photon, and its movement through absolute Euclidean space and absolute Newtonian time.

My approach to formulate an equation of motion for a photon moving through a gravitational field is analogous to the approach used to formulate an equation of motion that describes, explains, and predicts the trajectory of an artillery shell by taking

the rotational as well as the translational motion of the projectile into consideration [85-90]. The ratio of rotational motion to translational motion of projectiles is not constrained by the equipartition theory. Consequently, the goal of ballistic research and artillery science is to find the rifling twist that is just sufficient to provide the rotational motion necessary to stabilize the projectile while minimizing the loss of translational kinetic energy. By contrast, I assume in developing the equation of motion that describes the trajectory of a photon through a gravitational field that the equipartition theorem is applicable to photons and that the rotational kinetic energy of a photon is equal to its translational kinetic energy.

## 2.2. Using dynamical photons to analyze the deflection of starlight

In Einstein's [40,91,92] theory of light, the mechanical properties of the quantum of light, including energy and momentum, were described quantitatively and completely with elegant simplicity by point-like properties of  $h\nu$  and  $h\nu/c$ , respectively. However the lack of any predicted internal structure of the photon limited one's ability to visualize optical processes in mechanical terms and this may have had the unintended consequence of obscuring many of the unsolved mysteries inherent in the wave-particle duality. Realizing the inadequacy of his theory of light, Einstein [93] wrote to Lorentz in 1909 stating that, “*I am not at all of the opinion that one should think of light as being composed of mutually independent quanta localized in relatively small spaces.*” While, it has been productive at first to treat atoms and the elementary particles that comprise them as ideal, point-like particles propelled by forces through empty space much like the earth is propelled around the sun, Fermi and Yang [94] considered the possibility that some particles may not be elementary. Here I consider the possibility that the photon is not an elementary particle, but a composite structure, as proposed by William Bragg, Louis de Broglie, Pascual Jordan, Max Born and others [95-124], with internal motions [37]. I propose that the energy is equipartitioned between each degree of freedom (Fig. 2).

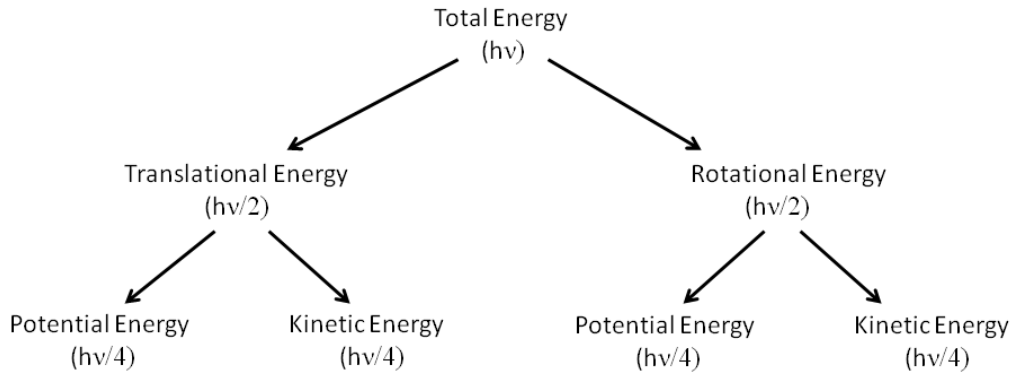


Fig.2: A model of the photon described in terms of the equipartition of energy. According to my model of a photon, the photon is composed of two complementary particles. The complementary particles form a harmonic oscillator that vibrates in the longitudinal direction, parallel to the propagation vector as it rotates orthogonally to the propagation vector [37]. Absorption consists of the elimination of the photon and a transfer of its total energy ( $h\nu$ ) to the absorber, while emission consists of the creation of a photon that results from the transfer of energy ( $h\nu$ ) from the emitter to the photon. The energy integral that describes the trajectory of a photon in a gravitational field makes use of the kinetic portion of the translational energy to describe the kinetic energy of the photon. By contrast, the gravitational energy of the photon used in the energy integral results from the interaction of the gravitational field of the sun with the total energy of the photon.

The total energy ( $E$ ) of a photon, which includes both translational energy and rotational energy, is given by the following equation:

$$E = h\nu \tag{2}$$

Where,  $h\nu$  is equal to the amount of energy required to create a photon during the emission process and is also the amount of energy transferred from a photon to matter during the absorption process. The linear momentum of a photon is given by  $\frac{h\nu}{c}$ . The relationship between the total energy of a photon and total momentum ( $p$ ) of a photon [125], as measured in processes in which the photon is absorbed, is:

$$p = \frac{E}{c} \tag{3}$$

When we define the momentum of a photon as a dynamical quantity given by the product of its apparent mass ( $m$ ) and its velocity ( $c$ ), we get:

$$p = mc \tag{4}$$

By equating Eqn. (3) and Eqn. (4), we get the well-known relationship between mass and energy [126]:

$$E = mc^2 \tag{5}$$

Solving for the apparent mass ( $m$ ) of a photon, we get:

$$m = \frac{p}{c} = \frac{E}{c^2} = \frac{h\nu}{c^2} \tag{6}$$

When starlight, composed of photons, passes near a massive body, it will be subjected to the gravitational binding energy of that body. I assume that the gravitational binding energy acts on the total mass-energy of the photon (Fig. 3). This assumption is supported by the agreement between theory and observation in my analysis of the gravitational red shift given in the next section. Following the thinking of Christiaan Huygens [127-130], who was working on describing and explaining the motion of a pendulum clock, the gravitational binding energy will cause a solid particle to be deflected in the radial direction toward the massive body instead of continuing in the tangential direction. If the translational kinetic energy of the photon is greater than the gravitational binding energy such that the total orbital energy of the photon is greater than zero ( $E_{orbital} > 0$ ), the photon will follow an unbounded and hyperbolic path around the massive body [131-133]. Indeed the word, hyperbola is derived from the Greek word,  $\upsilon\pi\epsilon\rho\beta\omicron\lambda\eta$ , which means overthrown. Consequently, the position of the star will appear to an observer to be displaced from its

actual position (Fig. 1). The displacement will depend in part on the relationship between the translational kinetic energy of the photon and the gravitational binding energy. The gravitational binding energy between a large gravitational mass

( $M$ ) and a photon with apparent mass ( $m$ ) separated by a center-to-center distance ( $r$ ) is given by:

$$E_{gravitational} = -\frac{GMm}{r} \quad (7)$$

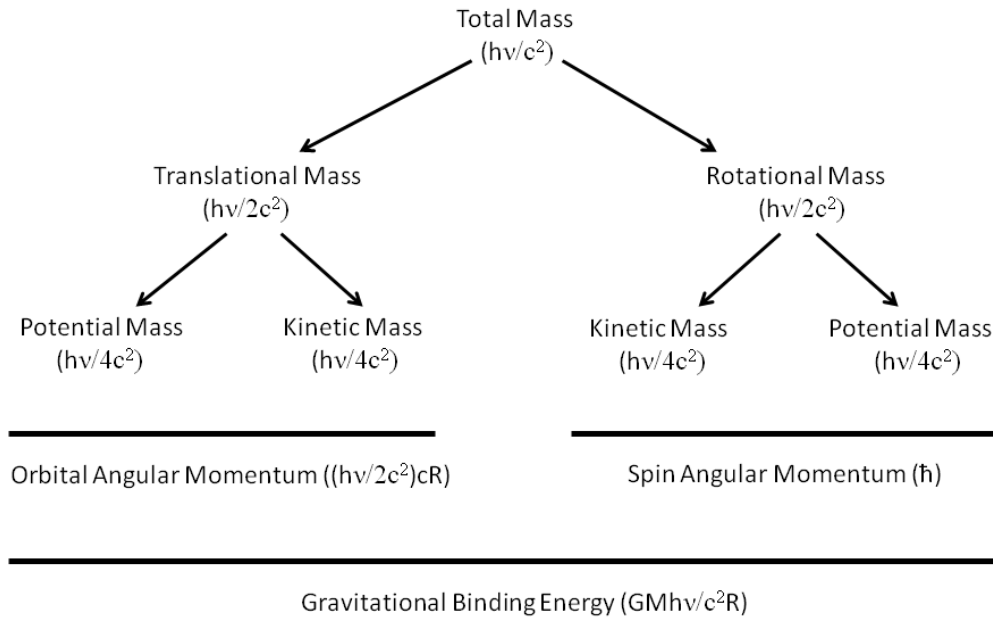


Fig.3: A model of the photon described in terms of equipartition of mass-energy. Using the mass-energy relation,  $m = \frac{E}{c^2}$ , given by Eqn. (6), and applying the equipartition theorem, the mass of the photon can be partitioned equally between the longitudinal harmonic oscillator and the rotational harmonic oscillator. I propose that the orbital angular momentum results from the translational mass and the spin angular momentum results from the rotational mass. As demonstrated for the gravitational red shift given in the next section, the gravitational potential interacts with the total mass of the photon.

Energy is conserved as the photon propagates through absolute Euclidean space and Newtonian time in its trajectory past a massive body. The energy integral of the orbital energy, which is a constant of motion that takes into consideration the translational kinetic energy ( $\frac{1}{4}mv^2$ ) of the photon and the gravitational binding energy ( $-\frac{GMm}{r}$ ) between the massive body and the photon is given by:

$$E_{orbital} = \frac{1}{4}mv^2 - \frac{GMm}{r} \quad (8)$$

Using polar coordinates and decomposing the translational kinetic energy into the radial ( $r$ ) and tangential ( $\theta$ ) components, we get:

$$E_{orbital} = \frac{1}{4}m\left(\frac{dr}{dt}\right)^2 + \frac{1}{4}mr^2\left(\frac{d\theta}{dt}\right)^2 - \frac{GMm}{r} \quad (9)$$

Eqn. (9) is a statement of the conservation of energy. As the photon passes a massive body, orbital angular momentum is also conserved. The orbital angular momentum of a photon following a hyperbolic trajectory as it approaches the sun is given in terms of its apparent mass, its velocity and the impact parameter, where the impact parameter is the perpendicular distance between the center of force ( $f$ , in Fig. 1) and the incident velocity [132]. The trajectory of the photon is restricted to a plane defined by the radius vector of the gravitational field and the translational velocity vector of the photon. Since there is no force perpendicular to this plane, the orbital angular momentum of the photon is conserved. Thus the orbital angular momentum forms an orbital angular momentum integral of the

trajectory of the photon, much like the orbital energy forms the energy integral of the trajectory. The two integrals, which are based on conservation of energy and conservation of angular momentum, respectively, act as adjustable parameters, which along with the initial conditions,  $r_o$  and  $\theta_o$ , yield a complete solution to the photon's trajectory in terms of the two degrees of freedom,  $r$  and  $\theta$ . The orbital angular momentum discussed here is not the same as the orbital angular momentum of light beams produced by lasers [134,135]. In the case for photons grazing the limb of the sun, as described here, the impact parameter, which is equivalent to the moment of inertia, is given by the radius of the sun,  $R$ .

I assume that only the translational mass, which is half of the total mass, contributes to the orbital angular momentum when a photon propagates in a trajectory around the sun (Fig. 3). The rotational motion of the photon, although present and ubiquitous, is a spinning motion and does not contribute to its orbital angular momentum. Since  $v = \left(\frac{d\theta}{dt}\right)r$ , the orbital angular momentum integral, which is a constant of motion based on the conservation of angular momentum, can be written like so:

$$L_{orbital} = \frac{1}{2}mvr = \frac{mr^2}{2} \left(\frac{d\theta}{dt}\right) \quad (10)$$

After rearranging Eqn. (10), we get:

$$\frac{d\theta}{dt} = \frac{2L_{orbital}}{mr^2} \quad (11)$$

Where,  $\frac{mr^2}{2}$  is the moment of inertia. After substituting Eqn. (11) into Eqn. (9), and cancelling like terms, Eqn. (9) can then be rewritten as:

$$E_{orbital} = \frac{1}{4}m \left(\frac{dr}{dt}\right)^2 + \left(\frac{L_{orbital}^2}{mr^2}\right) - \frac{GMm}{r} \quad (12)$$

After solving for  $\frac{dr}{dt}$ , we get:

$$\frac{dr}{dt} = \mp \sqrt{\frac{4E_{orbital}}{m} - \left(\frac{4L_{orbital}^2}{m^2r^2}\right) + \frac{4GM}{r}} \quad (13)$$

We can eliminate the time dependence inherent in the energy integral and the angular momentum integral given in Eqns. (13) and (11), respectively, by using the chain rule to combine these equations. This gives us the equation for the shape of the trajectory in terms of the change in the polar angle with respect to the change in the radial distance:

$$\frac{d\theta}{dr} = \frac{d\theta}{dt} \frac{dt}{dr} = \mp \frac{2L_{orbital}}{mr^2 \sqrt{\frac{4E_{orbital}}{m} - \left(\frac{4L_{orbital}^2}{m^2r^2}\right) + \frac{4GM}{r}}} \quad (14)$$

In order to integrate Eqn. (14), we separate the variables and simplify:

$$\int d\theta = \mp \int \frac{\left(\frac{L_{orbital}}{r}\right)^2 \left(\frac{1}{L_{orbital}}\right)}{\sqrt{\left[mE_{orbital} - \left(\frac{L_{orbital}}{r}\right)^2 + \frac{GMm^2}{r}\right]}} dr \quad (15)$$

We can conveniently integrate Eqn. (15) after substituting  $u = \frac{L_{orbital}}{r}$ , and simplifying:

$$\theta(r) - \theta_o = \pm \int \frac{du}{\sqrt{\left[-u^2 + \frac{GMm^2}{L_{orbital}}u + mE_{orbital}\right]}} \quad (16)$$

Where,  $\theta_o$  is the constant of integration. This integral can be solved using the following formula from a Table of Integrals:

$$\pm \int \frac{du}{\sqrt{[-au^2 + bu + c]}} = \pm \frac{1}{\sqrt{-a}} \sin^{-1} \left[ \frac{2au + b}{\sqrt{b^2 - 4ac}} \right] \quad (17)$$

Where,  $a = -1$ ,  $b = \frac{GMm^2}{L_{orbital}}$  and  $c = mE$ , and we take the negative solution<sup>2</sup> to yield the concave portion of the hyperbola relative to the origin and evaluated from 0 to  $\pi$  as shown in Fig. 1. After substituting the values for a, b, and c into Eqn. (17), we get:

$$\theta(r) = \theta_o - \sin^{-1} \left[ \frac{-\frac{L_{orbital}}{r} + \frac{GMm^2}{L_{orbital}}}{\sqrt{\left(\frac{GMm^2}{L_{orbital}}\right)^2 + 4mE_{orbital}}} \right] \quad (18)$$

After taking the sine of both sides, we get:

$$\sin(\theta) = \sin(\theta_o) - \frac{\frac{L_{orbital}}{r} - \frac{GMm^2}{L_{orbital}}}{\sqrt{\left(\frac{GMm^2}{L_{orbital}}\right)^2 + 4mE_{orbital}}} \quad (19)$$

Because  $\sin(0) = 0$ , by setting  $\theta_o = 0$ , we get:

$$\sin \theta = \frac{\frac{L_{orbital}}{r} - \frac{GMm^2}{L_{orbital}}}{\sqrt{\left(\frac{GMm^2}{L_{orbital}}\right)^2 + 4mE_{orbital}}} \quad (20)$$

<sup>2</sup> The positive solution applies to the concave trajectory of starlight on the opposite side of the sun.

After rearranging, we get:

$$\frac{L_{orbital}}{r} = \frac{GMm^2}{L_{orbital}} + \sqrt{\left(\frac{GMm^2}{L_{orbital}}\right)^2 + 4mE_{orbital} \sin \theta} \quad (21)$$

Next we rewrite Eqn. (21) in order to get  $r$  as a function of  $\theta$ :

$$r = \frac{\frac{L_{orbital}^2}{GMm^2}}{1 + \sqrt{1 + \frac{4mL_{orbital}^2 E_{orbital} \sin \theta}{G^2 M^2 m^4}}} \quad (22)$$

Eqn. (22) has the form of an equation for a conic section where one focus is at the origin. The focus is placed at the origin of the polar coordinate system because this point is described by the unique characteristic that every point in the trajectory taken by the photon is attracted towards the focal point by a force inversely proportional to the square of the distance between the focal point and the photon. The utility of the equation for a conic section comes from its ability to transform the characterization of the deflection of starlight from the polar coordinate system where the sun is at the center to a coordinate system of the observer where the sun is at the focus. In the initial condition, when  $\theta = \theta_o = 0$ ,  $r = r_o = \infty$ . When the energy integral,  $E_{orbital} > 0$ , and the eccentricity ( $\epsilon$ ) or degree of spread,  $\epsilon > 1$ , the equation describes a hyperbola in polar coordinates where:

$$r = \frac{\alpha}{1 + \epsilon \sin \theta} \quad (23)$$

and where  $\alpha$  is the semi-latus rectum, which is one half of the length of the chord passing through the focus and parallel to the directrix. The semi-latus rectum is equal to  $\frac{L_{orbital}^2}{GMm^2}$ . The magnitude of the semi-latus rectum is inversely proportional to the magnitude of the gravitational force. When  $\theta = \frac{\pi}{2}$ ,  $r$  intersects the ordinate at  $r = \frac{\alpha}{1 + \epsilon}$ . By comparing Eqn. (22) with Eqn. (23), we see that eccentricity ( $\epsilon$ ) is given by:

$$\epsilon = \sqrt{1 + \frac{4mL_{orbital}^2 E_{orbital}}{G^2 M^2 m^4}} \quad (24)$$

$E_{orbital}$  and  $L_{orbital}$  are constants of integration. By letting  $L_{orbital} = \frac{1}{2}mvr = \frac{mcR}{2}$ , where  $v = c$ , the speed of light, and  $r = R$ , the radius of the sun, we get:

$$\epsilon = \sqrt{1 + \frac{4mm^2 c^2 R^2 E_{orbital}}{4G^2 M^2 m^4}} = \sqrt{1 + \frac{c^2 R^2 E_{orbital}}{G^2 M^2 m}} \quad (25)$$

By letting  $E_{orbital} = \frac{1}{4}mv^2 - \frac{GMm}{r} = \frac{1}{4}mc^2 - \frac{GMm}{R}$ , where  $v = c$ , the speed of light, and  $r = R$ , the radius of the sun, we get:

$$\begin{aligned} \epsilon &= \sqrt{1 + \frac{c^2 R^2 mc^2}{4G^2 M^2 m} - \frac{c^2 R^2 GMm}{G^2 M^2 mR}} \\ &= \sqrt{1 + \frac{c^4 R^2}{4G^2 M^2} - \frac{c^2 R}{GM}} \end{aligned} \quad (26)$$

Where,  $\frac{c^4 R^2}{4G^2 M^2} = 5.5454936 \times 10^{10}$  and  $\frac{c^2 R}{GM} = 4.709774353 \times 10^5$ . Since  $\frac{c^4 R^2}{4G^2 M^2} \gg \frac{c^2 R}{GM}$  and  $\frac{c^4 R^2}{4G^2 M^2} \gg 1$ ,

$$\epsilon \cong \sqrt{\frac{c^4 R^2}{4G^2 M^2}} \cong \frac{c^2 R}{2GM} \quad (27)$$

After taking the reciprocal, we get:

$$\frac{1}{\epsilon} \cong \frac{2GM}{c^2 R} \cong 4.246487942 \times 10^{-6} \quad (28)$$

The final formula is independent of the mass of the photon, indicating that the gravitational deflection of starlight should not be a source of chromatic aberration. Thus the gravitational cause of the bending of starlight by the sun can be distinguished from the refraction of starlight by the radial gradient in refractive index that exists from the sun. The independence of the final formula on the mass of the photon further helps to justify the assumption, implicit in the derivation, that the mass of the photon remains constant. From the properties of a conic section, we can obtain  $\beta$ :

$$\beta = \cos^{-1} \left(\frac{1}{\epsilon}\right) \cong 89.99975669^\circ \quad (29)$$

Given that one degree equals 3600 arcseconds, we can obtain the predicted angle of deflection ( $\delta$ ) from  $\beta$  given in Eqn. (29) and from the relations shown in Fig. 1:



$$\delta \cong 180^\circ - 2\beta \cong 4.86612 \times 10^{-4\circ} \cong 1.75 \text{ arcseconds} \quad (30)$$

which is the same as the value of the “double deflection” predicted by Einstein’s General Theory of Relativity and observed by astronomers [22-24]. The generalized energy and angular momentum integrals, for a generalized photon propagating through the gravitational field of the sun, are given by:

$$E_{orbital} = \frac{1}{2N}mv^2 - \frac{GMm}{r} \quad (31)$$

and

$$L_{orbital} = \frac{1}{N}mvr = \frac{m}{N}\left(\frac{d\theta}{dt}\right)r^2 \quad (32)$$

Where, N characterizes the assumptions used to equipartition the mass-energy of the photon. N = 1 for a simple corpuscular photon with translational motion only, and N = 2 for a complex photon with translational and rotational motion. The predicted deflection for a simple Newtonian corpuscle that lacks rotational motion is given in Appendix 1. Using this derivation, the predicted deflection for a Newtonian corpuscle that lacks rotational motion is calculated to be equal to one-half the deflection calculated for a photon whose mass-energy is equipartitioned between its translational and rotational oscillating components. While my analysis leaves us ignorant of the physical mechanism by which the gravitational force acts between the sun and the photon [136,137], any putative physical mechanism is no less mysterious than the physical mechanism that must be imagined to explain how matter can warp a dynamic space-time continuum.

The correspondence between the predicted magnitude of the “double deflection” of starlight based on the assumptions of a complex and dynamic photon and the observed results supports the validity of the assumption of a complex and dynamical photon moving through absolute Euclidean space and Newtonian time. While I have used the composite nature of the photon to derive the observed gravitational deflection of starlight by the sun, I shall now use the observed deflection of starlight by the sun to refine my model of the photon [37], which did not take advantage of the constraints provided on the structure of a photon by the equipartition theorem. If we consider the observed gravitational deflection of starlight by the sun to be evidence for the equipartition of mass-energy between the translational and rotational

harmonic oscillators of the photon, then, the values of the radius (r) and the geometrical cross sectional area (σ) of the photon [1,37] must be revised. Given that the spin angular momentum of a photon is equal to ħ, in previous publications, I calculated the radius and geometrical cross section of the photon to be  $\frac{\lambda}{2\pi}$  and  $\frac{\lambda^2}{4\pi}$ , respectively, without taking into consideration that only one-half of the total mass-energy of the photon participates in the spin angular momentum [1,37]. Thus the revised radius of the photon is calculated from the following equation:

$$\hbar = \frac{mr^2}{2}\omega = \frac{\hbar\omega r^2}{2c^2}\omega \quad (33)$$

Where, ω is the angular frequency of the rotational harmonic oscillator. Solving for the square of the radius, we get:

$$r^2 = \frac{2c^2}{\omega^2} \quad (34)$$

Since  $c = \frac{\omega}{k}$  within a given inertial frame, Eqn. (34) becomes:

$$r^2 = \frac{2}{k^2} \quad (35)$$

Where, k is the wave number of the photon, and

$$r = \frac{\sqrt{2}}{k} \quad (36)$$

Since  $k = \frac{2\pi}{\lambda}$ , where λ is the wavelength of the photon,

$$r = \frac{\sqrt{2}\lambda}{2\pi} \quad (37)$$

and the refined estimate of the radius of the photon is  $\sqrt{2}$  times greater than the previous estimate [1,37]. Given that the geometrical cross sectional area is equal to  $\pi r^2$ , by taking the equipartition of mass-energy into consideration, the geometrical cross sectional area of a photon will be:

$$\sigma = \pi\left(\frac{\sqrt{2}\lambda}{2\pi}\right)^2 = \frac{\lambda^2}{2\pi} \quad (38)$$

and the refined estimate of the geometrical cross sectional area of a photon is two times greater than the previous estimate [1,37]. Consequently, the dissipative, optomechanical Doppler force, which opposes the acceleration of particles with a charge and/or a magnetic moment, and will give a

preferred direction to the arrow of time, will be twice as large as that previously calculated [1,69].

The theory to explain the gravitational deflection of starlight proffered here, which is based on a model of a complex and dynamic photon with translational and rotational motion propagating through absolute Euclidean space and Newtonian time, is equally applicable in describing and explaining gravitational lensing, where a massive body such as a galaxy or a black hole between an observer and a distant source such as a quasar results in the gravitational deflection of light [138-143]. Heretofore, gravitational lensing has been viewed exclusively as a confirmation of the effect of matter in curving a dynamic space-time continuum.

### 2.3. Using dynamical photons to analyze the gravitational red shift

Using Newton’s Law of Universal Gravitation to describe and explain the action of gravity on light, Priestley [31], Michell [32], and Laplace [33] postulated that the corpuscles of light emanating from a fixed star would be gravitationally attracted to the star in the same manner that any other body would be attracted to a gravitational mass. They did not assume, in their dynamical theories, that the speed of light in a vacuum was constant and surmised that overcoming the gravitational attraction would result in a diminution of the velocity of light. On the other hand, Einstein assumed that atoms were fundamentally clocks and that gravity was the apparent effect of matter warping a dynamical space-time continuum that surrounded the atomic clock. Consequently, the frequency of light emitted by an atom would be a function of the matter-dependent tilt of the space-time continuum in the location in which the light-emitting atom existed. In contrast to Einstein, I assume along with Newton [136] that time is absolute, and in contrast to Priestley, Michell, and Laplace, I assume that the speed of light ( $c$ ) in a vacuum is invariant and not affected by gravity. I assume that the invariant speed of light is exclusively a result of the properties of the vacuum through which it moves [144], and is characterized by the electrical permittivity ( $\epsilon_0$ ) and the magnetic permeability ( $\mu_0$ ):

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \tag{39}$$

By contrast, I assume that the frequency ( $\nu$ ) and the wavelength ( $\lambda$ ) of an emitted photon, the product of which equals  $c$ , are both affected by

gravity in such a way that their product remains invariant:

$$\nu \lambda = c \tag{40}$$

The gravitational potential energy between a gravitational mass ( $M$ ) and a dynamic photon, being emitted with apparent mass ( $m$ ), and separated from the gravitational mass by a center-to-center distance ( $R$ ), is given by:

$$E_{gravitational} = -\frac{GMm}{R} \tag{41}$$

Where,  $G$  is the gravitational constant and is equal to  $6.67300 \times 10^{11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ . Since the mass of a photon would have cancelled from both sides of any equation that described Newtonian mechanics, the value for the mass of a photon would have been unknown to Newton. We, however, can express Eqn. (41) in terms of the effect of gravity on a dynamic photon by using the definition of the apparent mass of a photon given in Eqn. (6):

$$E_{gravitational} = -\frac{GMh\nu_\infty}{Rc^2} \tag{42}$$

I assume that during emission by an atom on the surface of the star, the energy of a photon is diminished by the work it must do to overcome the gravitational binding energy of the star in order to reach an observer antiparallel to the radial gravitational vector<sup>3</sup>. While the total energy ( $E(\infty)_{photon}$ ) of a photon emitted by an atom an infinite distance from the star would be  $(h\nu_\infty)$ , the total energy ( $E(R)_{photon}$ ) of the photon emanating from a gravitational mass when it is at distance  $R$  relative to the center of the gravitational mass is given by  $h\nu_R$ . The difference in the energy of the photon when it is emitted in a gravitational field compared with one emitted in the absence of a gravitational field is given by the following equation:

<sup>3</sup> Since stars can be considered to be an infinite distance from the earth, here we are only discussing a photon emitted antiparallel to the radial gravitational vector by an atom in the star. Generally speaking, the magnitude of the gravitational diminution will depend quantitatively on the cosine of the angle between the  $k$  vector of the photon and the radial gravitational vector.

$$E(R)_{\text{photon}} = E(\infty)_{\text{photon}} + E_{\text{gravitational}}$$

$$= h\nu_{\infty} - \frac{GMh\nu_{\infty}}{Rc^2} \tag{43}$$

$$h\nu_R = h\nu_{\infty} - \frac{GMh\nu_{\infty}}{Rc^2} = h\nu_{\infty} \left( 1 - \frac{GM}{Rc^2} \right) \tag{44}$$

The energy integral for the emission of a photon in a gravitational field, which consists of the total energy of a photon emitted at infinity and the gravitational binding energy at  $R$ , is given in Eqn. (44). It is the adjustable parameter, which along with a boundary condition ( $r = \infty$ ), yields a complete solution to the frequency of a photon emitted in a gravitational field. We can solve for the gravity-induced energy shift in the emitted photon by rearranging Eqn. (44) and cancelling Planck's constant:

$$\frac{\nu_{\infty} - \nu_R}{\nu_{\infty}} = \frac{GM}{Rc^2} \tag{45}$$

The frequency of the emitted photon does not only depend on the position of the emitting atom in a gravitational field, but also on the direction of emission since the gravitational binding energy subtracts from the energy of a photon emitted in the direction pointing away from the gravitational mass and adds to the energy of a photon emitted in the direction pointing towards the gravitational mass (Fig. 4).

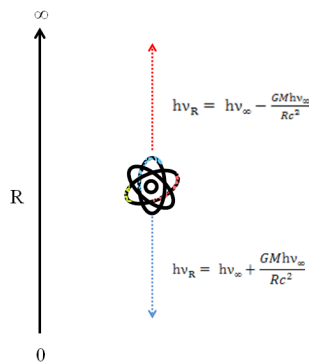


Fig.4: The frequency of the emitted photon depends on the direction of emission since the gravitational binding energy subtracts from the energy of a photon emitted in the direction pointing towards infinity, away from the gravitational mass. The photon is red shifted. The gravitational binding energy adds to the energy of a photon emitted in the direction pointing towards the center (0) of the gravitational mass. The photon is blue shifted.

The formula for the gravitational red shift presented in Eqn. (45), which is based on the apparent mass of a photon emitted by an atom in absolute Euclidean space and Newtonian time, Newton's Law of Universal Gravitation, and the conservation of energy, is indistinguishable from the formula for the gravitational red shift given by Einstein [13,145-151], which is based on the assumption of the influence of matter on warping a dynamical space-time continuum through which a point-like photon submissively propagates.

Since Eqn. (45), which is not based on the assumption of a point-like photon moving submissively through a dynamical space-time continuum warped by matter, describes the observed gravitational red shift [152-156], then the observed gravitational red shift is not exclusively and absolutely a confirmation of the influence of matter in warping a dynamical space-time continuum, but is by the same token also a confirmation of a dynamic photon, whose mass is equal to  $m = \frac{h\nu}{c^2}$ , moving through absolute Euclidean space and Newtonian time. Thus the relativity of a dynamic space-time continuum as postulated by the General Theory of Relativity is sufficient, but not necessary, to explain the observed gravitational red shift.

In a pamphlet entitled, **Introduction to Outer Space** [157], written by President Dwight Eisenhower's Science Advisory Committee, it is stated that *"Physicists are anxious to run one crucial and fairly simple gravity experiment as soon as possible. This experiment will test an important prediction made by Einstein's General Theory of Relativity, namely, that a clock will run faster as the gravitational field around it is reduced. If one of the fantastically accurate clocks, using atomic frequencies, were placed in a satellite and should run faster than its counterparts on earth, another of Einstein's great and daring predictions would be confirmed. (This is not the same as the prediction that any moving clock will appear to a stationary observer to lose time—a prediction that physicists already regard as well confirmed.)"*

This pamphlet stimulated terrestrial tests of the gravitational red shift predicted by Einstein's General Theory of Relativity. Robert Pound and associates [158-160] performed these tests that confirmed the predictions of Einstein's General Theory of Relativity concerning the gravitational red shift, and set the stage for the synchronization of clocks necessary for a Global Positioning System (GPS). According to David Mermin [151],

Einstein’s “general theory of relativity, which has become of fundamental importance in cosmology, in astrophysics, and even—remarkably for a subject that was long thought to be of only intellectual interest—in the very practical matter of how the global positioning system (GPS) operates here on Earth...his discovery [is] (of crucial importance for the GPS) that gravity affects the rate at which a clock runs....” Neil Ashby [161] states the relationship between Einstein’s General Theory of Relativity and the Global Positioning System even more strongly, “The GPS system is, in effect, a realization of Einstein’s view of space and time.”

Eqn. (45), which was obtained by considering the effect of gravitational binding energy on the energy of an emitted photon, is applicable for calculating the correction needed to synchronize the atomic clocks used in the Global Positioning System (GPS) when they are at various distances from the center of the earth. The correction necessary to calculate the frequency shift when one atomic clock is on the surface of the earth, where  $R_{\text{earth}}$  is the radius of the earth, and the other atomic clocks are on satellites a distance  $R_{\text{satellite}}$  from the center of the earth is given by the following equations:

$$\frac{GM}{c^2} \left( \frac{1}{R_{\text{earth}}} - \frac{1}{R_{\text{satellite}}} \right) = \frac{v_{\text{satellite}} - v_{\text{earth}}}{v_{\text{satellite}}} \quad (46)$$

$$\frac{-GM}{c^2} \left( \frac{1}{R_{\text{earth}}} - \frac{1}{R_{\text{satellite}}} \right) = \frac{v_{\text{earth}} - v_{\text{satellite}}}{v_{\text{satellite}}} \quad (47)$$

Thus the correction needed to synchronize the atomic clocks used in the Global Positioning System (GPS) can be determined by combining the concept of a dynamic photon whose mass is equal to  $m = \frac{h\nu}{c^2}$  with Newton’s Law of Universal Gravitation and using conservation of energy to determine the influence of gravity on the energy content of the emitted photon (Fig. 5). Eqn. (46) shows that the gravitational binding energy adds to the energy of a photon emitted towards the earth by an atomic clock on a satellite. Consequently, the frequency of the photon emitted by an atomic clock in the direction of the earth is greater than the frequency of a photon emitted by an atomic clock on earth in the direction of the satellite. By contrast, Eqn. (47) shows that the gravitational binding energy subtracts from the energy of the photon emitted by an atomic clock towards the satellite by an atomic clock on earth. As a result, the frequency of the emitted photon is less than the frequency of a photon emitted by an atomic clock

on a satellite in the direction of the earth. I conclude that an understanding of the gravitational red shift and the synchronization of clocks for the Global Positioning System does not come from the assumption that time is relative and depends on the position in a gravitational field that is warped by matter but that the energy of an emitted photon depends on its position in a gravitational field existing in absolute Euclidean space and Newtonian time.

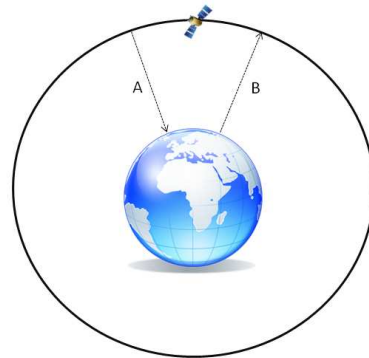


Fig.5: The Global Positioning System (GPS). (A) The gravitational binding energy increases the energy of a photon emitted towards the earth by an atomic clock on a satellite orbiting the earth. (B) The gravitational binding energy decreases the energy of a photon emitted towards the satellite by an atomic clock on the earth. *Caeteris paribus*, the frequency of the signal traveling from the satellite to the earth is greater than the frequency of the signal traveling from the earth to the satellite. Likewise, the period or clock rate of the signal traveling from the satellite to the earth is shorter than the period or clock rate of the signal traveling from the earth to the satellite.

While it is commonly believed that since the Global Positioning System uses the equations of Einstein’s relativity theories to synchronize the clocks, then space-time itself, as proffered by Einstein’s Theories of Relativity, must be relative, curved, and bendable [162-164], I have shown, it is possible to derive the formula for the synchronization of atomic clocks used in the Global Positioning System without invoking the assumption of the relativity of space-time. Therefore, the General Theory of Relativity is sufficient, but not necessary, for the synchronization of atomic clocks utilized for the functioning of Global Positioning System, and the GPS system is not necessarily a realization of Einstein’s view of space and time as a dynamic space-time continuum.

### 3. Conclusion

The gravitational deflection of starlight by the sun observed by the expeditions sent out jointly by the Royal Society of London and the Royal Astronomical Society on May 29, 1919 is considered to be one of the crucial and the most dramatic tests of Einstein’s General Theory of Relativity [165,166]. The astonishing and extraordinary nature of the confirmation was captured by John Burdon Sanderson Haldane [167], who wrote, “I do not doubt that he [Einstein] will be believed. A prophet who can give signs in the heavens is always believed....Einstein has told us that space, time, and matter are shadows of the fifth dimension, and the heavens have declared their glory.” Nevertheless, here I show, that by taking the known and reasonably proposed dynamical properties of photons into consideration, the gravitational deflection of starlight and the gravitational red shift can be explained in terms of absolute Euclidean space and Newtonian time without invoking the relativity of time and space. I suggest that the scientific evidence thought to solely, exclusively and indubitably support the relativity of space and time proffered by Einstein’s General Theory of Relativity is not as strong as we have heretofore assumed. Perhaps it is time to question the foundational nature of the General Theory of Relativity. Such questioning may be useful for unifying gravitational theory at the cosmic scale with quantum theories at the subatomic scale.

#### Acknowledgments

I am thankful for the reviewers whose suggestions substantially improved this paper.

#### Appendix A

##### Scattering of a Newtonian Corpuscle Without Rotational Energy

Unlike the composite photon modeled above, for a Newtonian corpuscle in free space, there is no rotational energy and thus the translational kinetic energy is equal to the total kinetic energy. The energy integral, which takes into consideration the translational kinetic energy ( $\frac{1}{2}mv^2$ ) of a Newtonian corpuscle and the gravitational binding energy ( $-\frac{GMm}{r}$ ) between the massive body and the photon is given by:

$$E_{orbital} = \frac{1}{2}mv^2 - \frac{GMm}{r} \quad (A1)$$

Using polar coordinates and decomposing the translational kinetic energy into the radial ( $r$ ) and tangential ( $\theta$ ) components, we get the equation for the conservation of energy:

$$E_{orbital} = \frac{1}{2}m\left(\frac{dr}{dt}\right)^2 + \frac{1}{2}mr^2\left(\frac{d\theta}{dt}\right)^2 - \frac{GMm}{r} \quad (A2)$$

The orbital angular momentum of a Newtonian corpuscle following a hyperbolic trajectory as it approaches the sun is conserved and is given in terms of its apparent mass, its velocity and the impact parameter, where the impact parameter is the perpendicular distance between the center of force ( $f$ , in Fig. 1) and the incident velocity. In the case of Newtonian corpuscles grazing the limb of the sun, as described here, the impact parameter is given by the radius of the sun,  $R$ . I assume that the translational mass, which is equal to the total mass of a Newtonian corpuscle, contributes to the orbital angular momentum (Fig. 3).

Since  $v = \left(\frac{d\theta}{dt}\right)r^2$ , the orbital angular momentum integral can be written like so:

$$L_{orbital} = mvr = mr^2\left(\frac{d\theta}{dt}\right) \quad (A3)$$

After rearranging Eqn. (A3), we get:

$$\frac{d\theta}{dt} = \frac{L_{orbital}}{mr^2} \quad (A4)$$

After cancelling like terms and combining Eqn. (A4) with (A2), Eqn. (A2) can be rewritten as:

$$E_{orbital} = \frac{1}{2}m\left(\frac{dr}{dt}\right)^2 + \left(\frac{L_{orbital}^2}{2mr^2}\right) - \frac{GMm}{r} \quad (A5)$$

After solving for  $\frac{dr}{dt}$ , we get:

$$\frac{dr}{dt} = \mp \sqrt{\frac{2E_{orbital}}{m} - \left(\frac{L_{orbital}^2}{m^2r^2}\right) + \frac{2GM}{r}} \quad (A6)$$

We can eliminate the time dependence inherent in the energy and angular momentum integrals given in Eqns. (A4) and (A6), respectively, by combining Eqns. (A4) and (A6). This gives us the equation for the shape of the trajectory in terms of the change in the polar angle with respect to the change in the radial distance:

$$\frac{d\theta}{dr} = \frac{d\theta}{dt} \frac{dt}{dr} = \mp \frac{L_{orbital}}{mr^2 \sqrt{\frac{2E_{orbital}}{m} - \left(\frac{L_{orbital}^2}{m^2r^2}\right) + \frac{2GM}{r}}} \quad (A7)$$

Then, in order to integrate Eqn. (A7), we separate the variables and simplify:

$$\int d\theta = \mp \int \frac{\left(\frac{L_{orbital}}{r}\right)^2 \left(\frac{1}{L_{orbital}}\right)}{\sqrt{\left[2mE_{orbital} - \left(\frac{L_{orbital}}{r}\right)^2 + \frac{2GMm^2}{r}\right]}} dr \tag{A8}$$

We can conveniently integrate Eqn. (A8) by substituting  $u = \frac{L_{orbital}}{r}$  and simplifying:

$$\theta(r) - \theta_o = \pm \int \frac{du}{\sqrt{\left[-u^2 + \frac{2GMm^2}{L_{orbital}}u + 2mE_{orbital}\right]}} \tag{A9}$$

Where,  $\theta_o$  is the constant of integration. This integral can be solved using a formula from a Table of Integrals

$$\pm \int \frac{du}{\sqrt{[-au^2 + bu + c]}} = \pm \frac{1}{\sqrt{-a}} \sin^{-1} \left[ \frac{2au + b}{\sqrt{b^2 - 4ac}} \right] \tag{A10}$$

Where,  $a = -1$ ,  $b = \frac{2GMm^2}{L_{orbital}}$  and  $c = 2mE$ , and we take the negative solution to yield the convex portion of the hyperbola relative to the origin and evaluated from 0 to  $\pi$  (Fig. 1). After substituting the values for a, b, and c into Eqn. (A10), we get:

$$\theta(r) = \theta_o - \sin^{-1} \left[ \frac{\frac{-2L_{orbital}}{r} + \frac{2GMm^2}{L_{orbital}}}{\sqrt{\left(\frac{2GMm^2}{L_{orbital}}\right)^2 + 8mE_{orbital}}} \right] \tag{A11}$$

After taking the sine of both sides, we get:

$$\sin(\theta) = \sin(\theta_o) - \frac{\frac{-2L_{orbital}}{r} + \frac{2GMm^2}{L_{orbital}}}{\sqrt{\left(\frac{2GMm^2}{L_{orbital}}\right)^2 + 8mE_{orbital}}} \tag{A12}$$

Because  $\sin(0) = 0$ , by setting  $\theta_o = 0$ , after rearranging, we get:

$$\frac{2L_{orbital}}{r} = \frac{2GMm^2}{L_{orbital}} + \sqrt{\left(\frac{2GMm^2}{L_{orbital}}\right)^2 + 8mE_{orbital}} \sin \theta \tag{A13}$$

Next we rewrite Eqn. (A13) to get  $r$  as a function of  $\theta$  and simplify:

$$r = \frac{\frac{L_{orbital}^2}{GMm^2}}{1 + \sqrt{1 + \frac{2mL_{orbital}^2 E_{orbital}}{G^2 M^2 m^4}} \sin \theta} \tag{A14}$$

Eqn. (A14) has the form of an equation for a conic section where one focus is at the origin. When  $E_{orbital} > 0$ , and  $\epsilon > 1$ , the equation describes a hyperbola where:

$$r = \frac{\alpha}{1 + \epsilon \sin \theta} \tag{A15}$$

By comparing Eqn. (A14) with Eqn. (A15), we see that eccentricity ( $\epsilon$ ) is given by:

$$\epsilon = \sqrt{1 + \frac{2mL_{orbital}^2 E_{orbital}}{G^2 M^2 m^4}} \tag{A16}$$

By letting  $E_{orbital} = \frac{1}{2}mc^2 - \frac{GMm}{R}$  and  $L_{orbital} = mcR$  for a Newtonian corpuscle, where  $c$  is the speed of light and  $R$  is the radius of the sun, we get:

$$\begin{aligned} \epsilon &= \sqrt{1 + \frac{2mm^2 c^2 R^2 E_{orbital}}{G^2 M^2 m^4}} \\ &= \sqrt{1 + \frac{c^2 R^2 mc^2}{G^2 M^2 m} - \frac{c^2 R^2 GMm}{G^2 M^2 mR}} \cong \frac{c^2 R}{GM} \end{aligned} \tag{A17}$$

Where,  $\frac{c^4 R^2}{G^2 M^2} = 2.21819744 \times 10^{11}$  and  $\frac{c^2 R}{GM} = 4.709774353 \times 10^5$ . Since  $\frac{c^4 R^2}{G^2 M^2} \gg \frac{c^2 R}{GM}$  and  $\frac{c^4 R^2}{G^2 M^2} \gg 1$ . After taking the reciprocal, we get:

$$\frac{1}{\epsilon} \cong \frac{GM}{c^2 R} \cong 2.123243971 \times 10^{-6} \tag{A18}$$

From the properties of a conic section, we can obtain  $\beta$ :

$$\beta = \cos^{-1} \left( \frac{1}{\epsilon} \right) \cong 89.99987835^\circ \tag{A19}$$

We can obtain the predicted angle of deflection ( $\delta$ ) for a Newtonian corpuscle from  $\beta$  given in Eqn. (A20) and from the relations shown in Fig. 1:

$$\begin{aligned} \delta &\cong 180^\circ - 2\beta \cong 2.43306 \times 10^{-4}^\circ \\ &\cong 0.8759 \text{ arcseconds} \end{aligned} \tag{A20}$$

Thus, using the same analysis we used to obtain the “double deflection” of a composite photon that has internal structure with both translational and rotational motion, we determined that if a photon had the properties of a Newtonian corpuscle that

had neither internal structure nor rotation, it would only give a “single deflection.” The observed deflection of starlight can be considered to be evidence for the complex, dynamic nature of a photon that moves through Euclidean space and absolute Newtonian time. Thus the “double deflection” observed by the astronomers on the eclipse expeditions can be explained by assuming that that photon is point-like and propagates through a dynamical space-time continuum that is warped by matter as posited by the General Theory of Relativity, or by assuming that space is absolute and Euclidean, time is absolute and Newtonian, and the photon has a complex dynamical structure with both translational and rotational motions.

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Received: 23 May 2012

Accepted: 13 June 2012